
Physics 599

STRINGS AND THE STRONG FORCE

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STRONG FORCE: Quantum Chromodynamics (QCD)

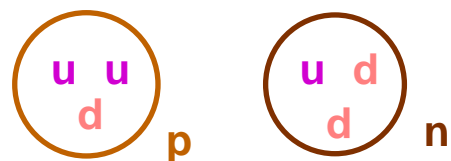
Quarks and gluons have colors

▶ cannot be free (directly detected)

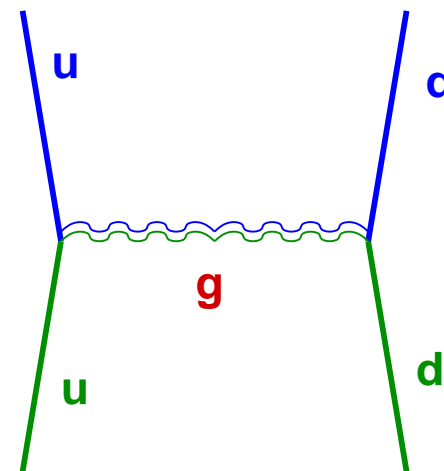
Only color neutral objects can be “seen”
(nucleons, mesons, etc)

Scale:

$$\Lambda_{QCD} = 0.236 \text{ GeV}$$



nucleons consist of quarks.



interaction via gluons.

★ asymptotic freedom

At high energies: coupling constant *decreases*

▶ spin-1 *charged* bosons (gluons) make vacuum paramagnetic

$$\alpha_s(E) = \frac{2\pi}{(11 - \frac{2}{3}N_f) \ln(E/\Lambda_{QCD})}$$

[Politzer; Gross and Wilczek - Nobel Prize 2004]

N_f : number of quarks.

“As time goes by, it seems that in fundamental physics we are circling around a particular set of themes and ideas. One of the important examples of such “limit cycles” is the relations between gauge fields and strings. The logic of these connections is as following. We begin with an observation (by K. Wilson) that in the strong coupling limit of a lattice gauge theory the elementary excitations are represented by closed strings formed by the color- electric fluxes. In the presence of quarks these strings open up and end on the quarks, thus guaranteeing quark confinement. More over, in the $SU(N)$ gauge theory the strings interaction is weak at large N . This fact makes it reasonable to expect that also in the physically interesting continuous limit (not accessible by the strong coupling approximation) the best description of the theory should involve the flux lines (strings) and not fields, thus returning us from Maxwell to Faraday. In other words it is natural to expect an exact duality between gauge fields and strings. The challenge is to build a precise theory on the string side of this duality.”

– A. M. Polyakov

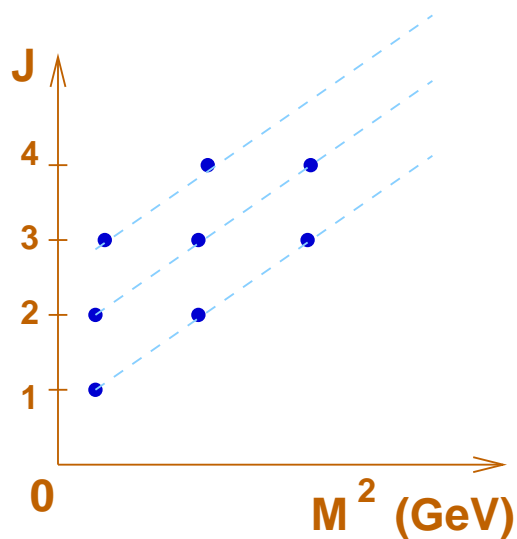
STRING THEORY vs QCD

Historically, string theory was invented as a theory of strong interactions.

In the 1960s, it was observed that hadronic masses were related to spin via

$$J = \alpha' M^2 + \alpha_0, \quad \alpha' \simeq (1 \text{ GeV}/c^2)^{-2}$$

(Regge trajectories; α' is Regge slope)



Spinning string has spin - mass relationship

$$J \sim M^2$$

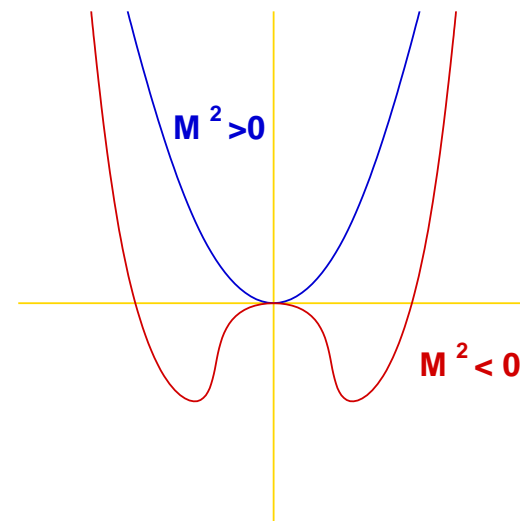
matching Regge trajectories of hadrons.

◆ Problem: tachyon, $M^2 < 0$ implies instability

♣ String theory was killed by QCD.

♡ Was resurrected as theory of gravity (with different scale,

$$m_P \simeq 10^{19} \text{ GeV}/c^2 \text{ instead of } m_{\text{proton}} \simeq 1 \text{ GeV}/c^2).$$



QUANTUM MECHANICS + GRAVITY

Newton's constant G is like weak (Fermi) constant G_F .
needs to be expressed in terms of a massive boson

$$G \sim \frac{g^2}{m_P^2}, \quad m_P = \sqrt{\frac{\hbar c}{G}} \simeq 10^{19} \text{ GeV}/c^2$$

P for Planck.

m_P is the scale where quantum effects are expected to become important.

QUESTION: What classical theory modifies Einstein's GR by including these massive bosons (in addition to the massless gravitational field)?

ANSWER: String theory!

KALUZA-KLEIN UNIFICATION

first hint at extra dimensions.



$$ds^2 = G_{AB} dx^A dx^B, \quad A, B = 0, 1, \dots, 4$$

$$x^A = (x^\mu, y)$$

$$0 = 2\pi R$$

extra dimension y forms circle of radius R .

Momentum $p_y = \hbar \frac{n}{R}$ ($n \in \mathbb{Z}$).

5d massless field $\Phi \Rightarrow$ 4d massive modes: $m_n^2 \equiv p_\mu p^\mu / c^2 = \frac{n^2 \hbar^2}{c^2 R^2}$

Parametrize metric (assume no y -dependence)

$$G_{AB} = \begin{pmatrix} g_{\mu\nu} + \Phi^2 A_\mu A_\nu & \vdots & e^{2\Phi} A_\mu \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ e^{2\Phi} A_\mu & \vdots & e^{2\Phi} \end{pmatrix}$$

5d Ricci scalar: $R = R_4 - \frac{1}{4} e^{2\Phi} F_{\mu\nu} F^{\mu\nu} - \partial_\mu \Phi \partial^\mu \Phi$

\Rightarrow 4d gravity + e&m + extra scalar.

(get rid of extra scalar by assuming it settles at its vacuum expectation value.)

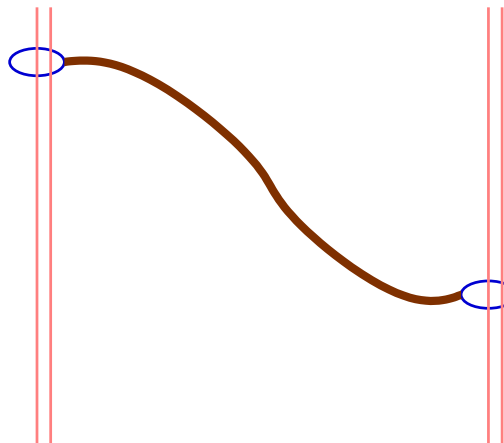
★ **WARNING:** NOT a quantum theory!

STRING THEORY

NON-RELATIVISTIC STRINGS:

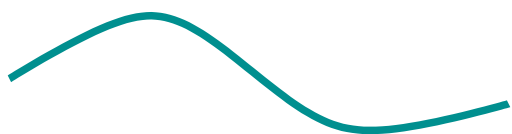


attached to D0-branes

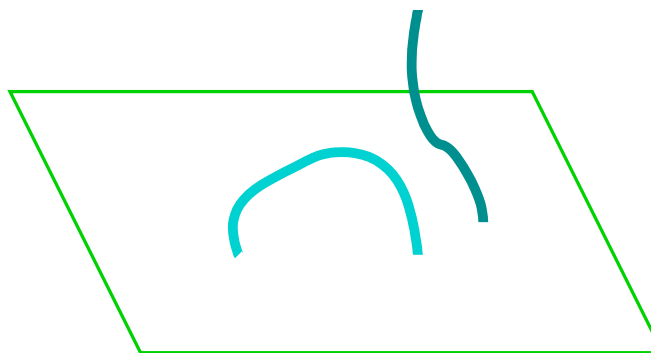


attached to D1-branes

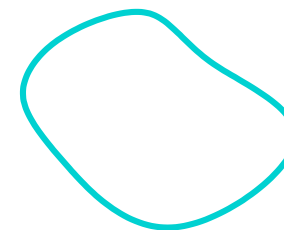
RELATIVISTIC STRINGS:



open (free)



attached to D2-brane



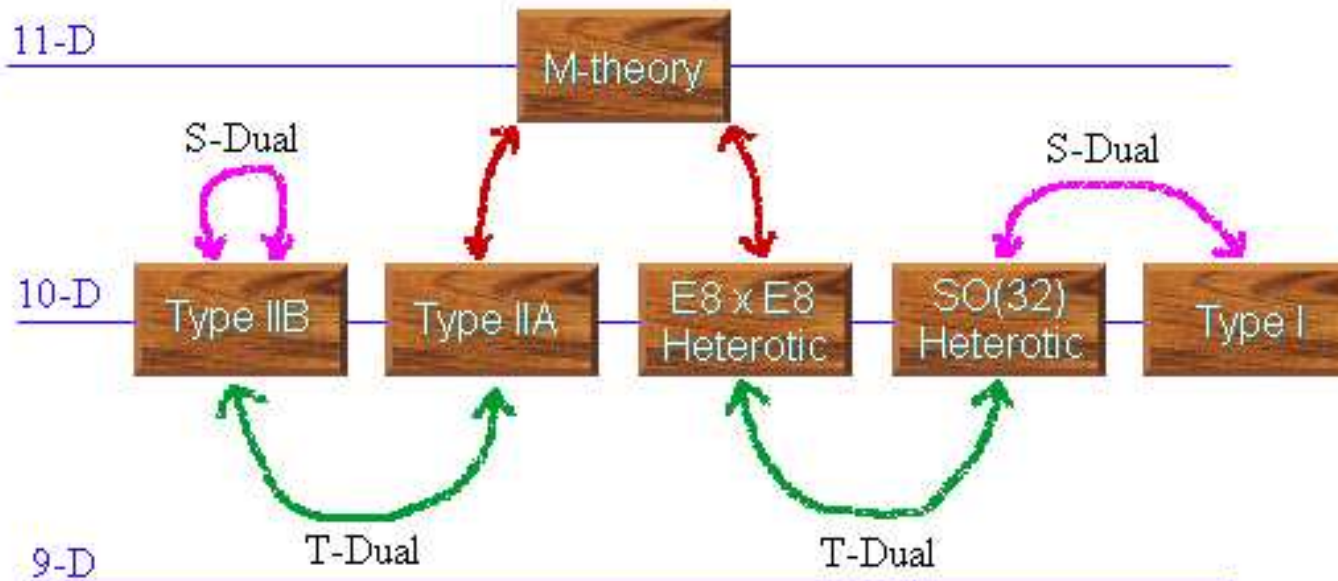
closed

★ modes: Fock space for (open) closed strings includes (photon) graviton.
 massive modes $\sim m_P$.

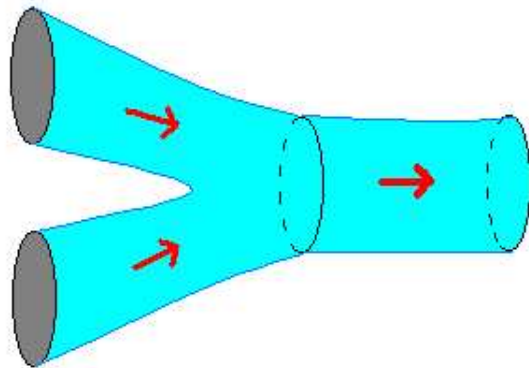
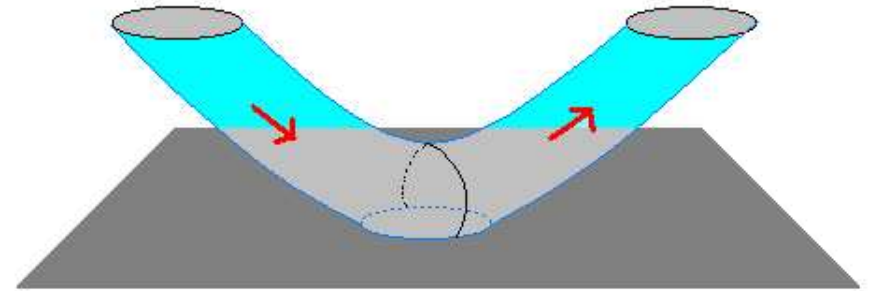
gauge symmetry: *reparametrization invariance*

► **anomalies** fix *spacetime dimension* and *gauge group*

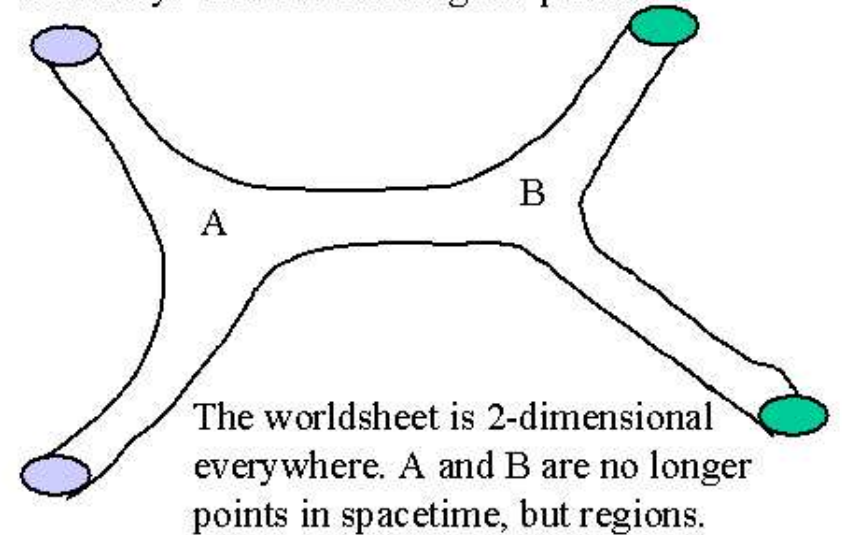
- bosonic strings in 26 dimensions - have tachyons
- superstrings in 10 dimensions - no tachyons



- ▶ string theory predicts
 - strings (particles)
 - D-branes (extended objects, solitons)
- ▶ interactions: splitting and joining



Strings interact by splitting and merging smoothly. There are no singular points.



EXTRA DIMENSIONS AND GRAVITY

$$F_{gr} = G \frac{m_{\text{proton}}^2}{r^2}, F_{el} = \frac{e^2}{4\pi r^2}$$



$$\frac{F_{gr}}{F_{el}} = \frac{1}{\alpha} \frac{m_{\text{proton}}^2}{m_P^2} \sim 10^{-36}$$

weak gravity $\because m_{\text{proton}} \ll m_P$ (*hierarchy* problem).

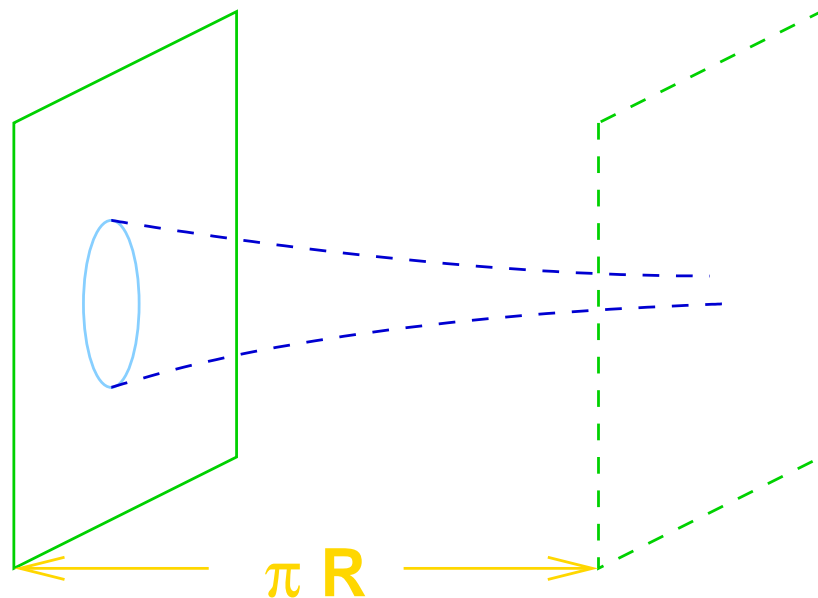
Randall-Sundrum scenario

include “warped” fifth dimension
(Anti de Sitter (AdS) space)

$$ds^2 = e^{-2ky} dx_\mu dx^\mu + dy^2$$

D3-branes at $y = 0, \pi R$

(we live at $y = 0$, other brane is regulator)



5d Planck scale M . Integrate over 5th dim \Rightarrow 4d (our) Planck scale:

$$m_P^2 = \frac{M^3}{m_k} (1 - e^{-2\pi kR}) \quad , \quad m_k = \frac{\hbar k}{c}$$

5d mass $m_0 \rightarrow$ 4d mass m , $m^2 = e^{-2\pi kR} m_0^2$

Can have $M \sim m_P \sim m_k \sim m_0$. If $kR \sim 50$, then $m \sim 1$ TeV.

► Solves hierarchy problem

\Rightarrow strong gravity effects at LHC!

Correction to Newtonian potential

$$V(r) = -\frac{GM}{r} \left\{ 1 + o\left(\frac{\ell_P^2}{r^2}\right) \right\}$$

tiny down to $r \sim \ell_P = \sqrt{\frac{G\hbar}{c^3}} \sim 10^{-33}$ cm.

★ In general, with extra dimensions:

- Particle theory \Rightarrow modification of both Coulomb and Newton's Laws.
- String theory \Rightarrow only Newton's Law is modified

modification of Newton's Law observed \Rightarrow signal of string theory



LHC at CERN, Geneva, Switzerland

STRINGS AND QCD (AGAIN)

QCD

- conformal in UV
- confining in IR
- spectrum of color singlets; scale

$$\Lambda_{QCD} = 0.263 \text{ GeV}$$

Vector mesons: momentum p^μ and internal orbital angular momentum L ;

► interpolating operator

$$\mathcal{O}_L^\mu = \bar{\psi} \gamma^\mu D_{\{\mu_1 \dots \mu_L\}} \psi$$

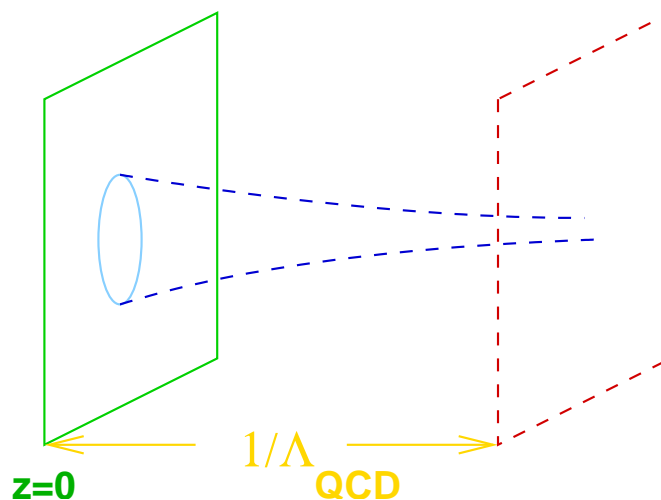
($\langle p | \mathcal{O}_L^\mu | 0 \rangle \neq 0$) consisting of quarks and gluons

AdS space (warped fifth dimension)

[de Téramond and Brodsky]

$$ds^2 = e^{-2ky} dx_\mu dx^\mu + dy^2$$

Let $z = \frac{1}{k} e^{ky}$. AdS boundary is at $z \rightarrow 0$.



bulk massless wave equation

$$z^2 f''_{\mu} - 3z f'_{\mu} + \{z^2 p^2 - (L+3)(L-1) + 3\} f_{\mu} = 0$$

Boundary condition:

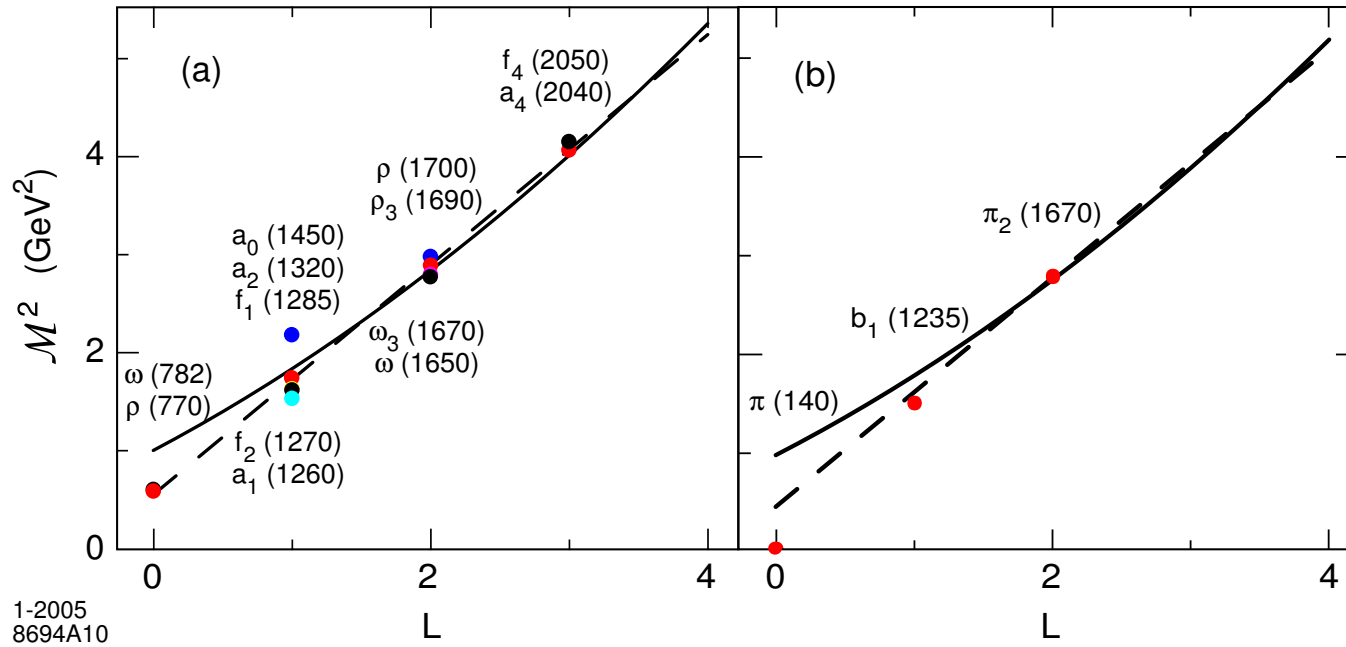
$$f_{\mu}(z) \Big|_{z=z_0} = 0, \quad z_0 = \hbar c / \Lambda_{QCD}$$

Eigenfunctions:

$$f_{\mu}(z) = e^{-ip \cdot x} z^2 J_{L+1}(z \beta_{L+1,k} / z_0), \quad J_{\nu}(\beta_{\nu,k}) = 0$$

Corresponding mass spectrum:

$$\mathcal{M}^2 = p^2 = \beta_{L+1,k}^2 \Lambda_{QCD}^2$$



Light meson orbital states. (a) vector mesons; (b) pseudoscalar mesons. The dashed line has slope 1.16 GeV² and is drawn for comparison.

Infinite extra dimensions

[Dvali-Gabadadze-Porrati (DGP) model]

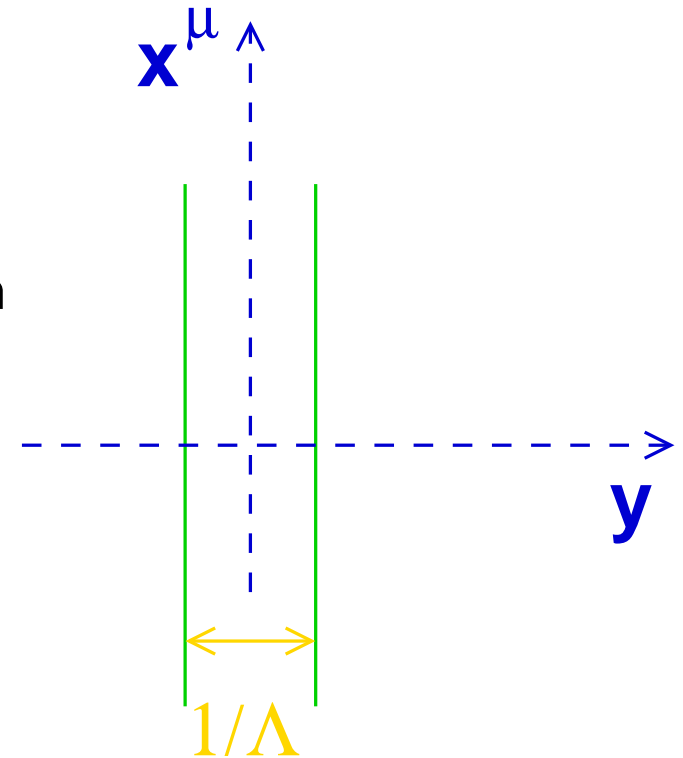
Scales:

- m_P (brane) and M (bulk) Planck scales,
- Λ (inverse transverse width of brane from quantum fluctuations)

modification of Newton's law

- at small distances
- at astronomically large distances

Problem: tachyonic modes for $D > 6$.



Infinite extra dimensions and QCD [GS]

D -dimensional bulk; M : bulk Planck mass. Spectrum:

$$p^2 = \beta_{L+\frac{D-8}{2},k}^2 m_0^2, \quad m_0^2 = \frac{M^{D-2}}{\omega_{D-4} m_P^2 \Lambda^{D-6}}$$

To match experimental data, require $D = 10$ and $m_0 = \Lambda_{QCD}$.

For $\Lambda \sim M$, we deduce

$$M \sim \sqrt{m_P \Lambda_{QCD}} \sim 10^9 \text{ GeV}$$

HIGH ENERGY SCATTERING

STRINGS

Regge limit ($s \rightarrow \infty$, t fixed - small angle)

$$\sigma \sim s^{2\alpha't+2}$$

Hard scattering limit ($s \rightarrow \infty$, t/s (angle) fixed)

$$\mathcal{A} \sim e^{-\alpha' s F(\theta)}, \quad F(\theta) = -\sin^2 \frac{\theta}{2} \ln \sin^2 \frac{\theta}{2} - \cos^2 \frac{\theta}{2} \ln \cos^2 \frac{\theta}{2}$$

soft! (smooth object of size $\sim \sqrt{\alpha'}$)

QCD

Regge limit (soft Pomeron behavior):

$$\sigma \sim s^\epsilon, \quad \epsilon = 0.0933 \pm 0.0024, \quad s \gtrsim 9 \text{ GeV}$$

Hard scattering limit:

$$\mathcal{A}_{2 \rightarrow m} \sim \frac{1}{s^\eta}, \quad \eta = \frac{1}{2} \sum_{i=1}^{m+2} \tau_i - 2$$

τ_i : twist = dimension – spin = number of constituents of i^{th} hadron.

STRINGS + BRANE [Polchinski and Strassler]

At z , momentum

$$\tilde{p}_\mu = \frac{z}{R} p_\mu$$

\Rightarrow observed momentum p_μ is *redshifted*.

Scalar (spin-0) wavefunction

$$\Phi = e^{ip \cdot x} f(z)$$

Near boundary ($z \rightarrow 0$),

$$f(z) \sim z^\Delta$$

Δ : dimension of operator creating state = τ (twist).

Scattering amplitude ($s = p^2$):

$$\mathcal{A}_{2 \rightarrow m}(p) = \int_0^\infty \frac{dz}{z^5} \mathcal{A}_{\text{string}}(\tilde{p}) \prod_{i=1}^{m+2} f_i(z) \quad , \quad \mathcal{A}_{\text{string}}(p) \sim e^{-\alpha' p^2 F(\theta)}$$

Integral dominated by $\alpha' \tilde{p}^2 \sim 1$, therefore

$$z \sim \frac{R}{\sqrt{\alpha' s}} \Rightarrow \mathcal{A}_{2 \rightarrow m}(p) \sim \frac{1}{s^\eta}, \quad \eta = \frac{1}{2} \sum_{i=1}^{m+2} \Delta_i - 2$$

in agreement with QCD!

FROISSART BOUND

under general assumptions, unitarity implies

$$\sigma \lesssim \frac{1}{M} \ln^2 s, \quad s \rightarrow \infty, \quad t = \text{fixed}$$

where M is mass of smallest excitation

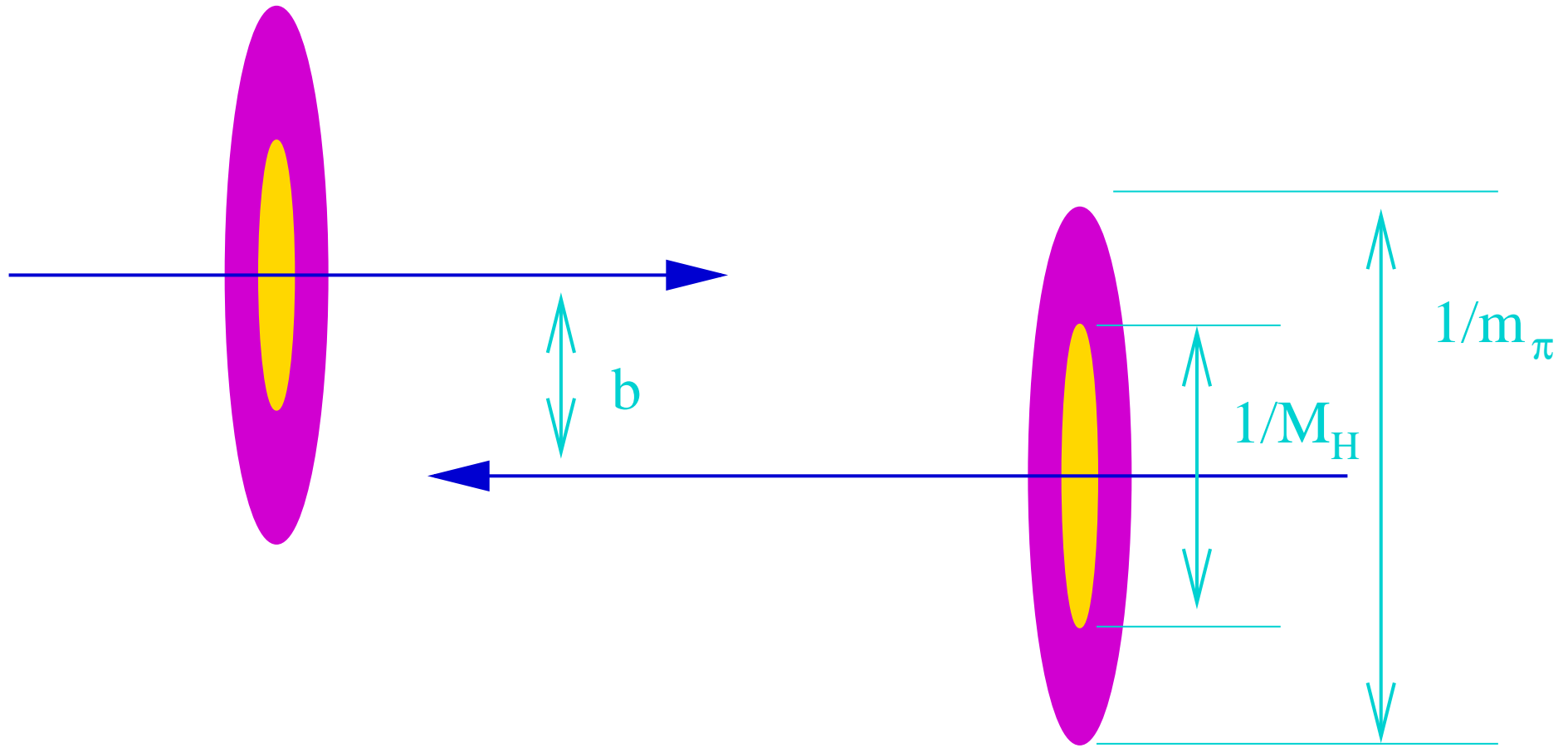
(in QCD, $M = m_\pi$, mass of almost Goldstone boson).

CHALLENGES:

- Prove bound in QCD.
 \hookrightarrow still open problem.
- Is bound saturated?
 \hookrightarrow was answered *before* QCD and Froissart bound were discovered!

HEISENBERG MODEL

Consider two colliding hadrons, size $\sim 1/M_H$ each.



Assumption: cloud of virtual π surrounds each hadron with radius $\sim 1/m_\pi$.

As $s \rightarrow \infty$, π clouds are shockwaves; hadron size (structure) irrelevant.

Scalar π (ignore isospin) described by a Dirac-Born-Infeld (DBI) action

$$S = \frac{1}{\ell^2} \int d^4x \sqrt{(\partial_\mu \phi)^2 + m_\pi^2 \phi^2 + 1/\ell^4}$$

where ℓ is a length scale.

RESULT:

$$\sigma \simeq \frac{\pi}{m_\pi^2} \ln^2 \frac{\sqrt{s}}{m_\pi}$$

saturating the Froissart bound.

STRINGS AND BRANES [Giddings]

IR brane at $z = z_0 = 1/\Lambda_{QCD}$ (hiding our ignorance of IR physics).

More precisely, choose $1/z_0 \sim$ lightest glueball mass, so *define*

$$\Lambda_{QCD} \sim 1 \text{ GeV}$$

5d (AdS) length scale

$$R = \frac{N_c^{1/4}}{M_P}$$

where N_c is number of colors (for QCD, $N_c = 3$).

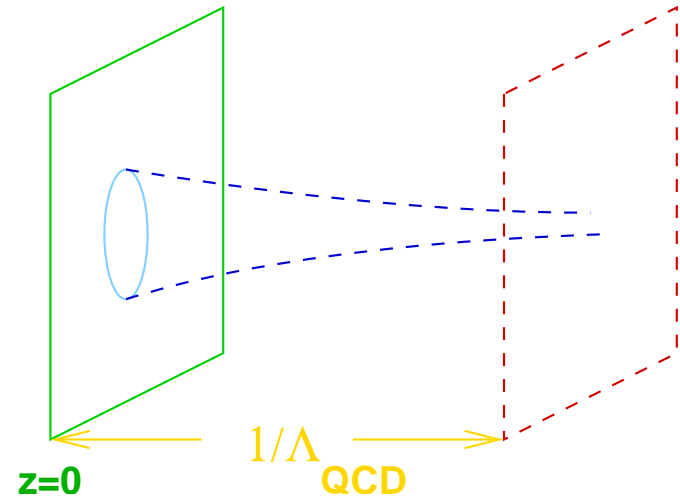
At Planck scale (M_P) \Rightarrow production of black holes.

Observed scale:

$$\hat{M}_P = \frac{R}{z_0} M_P = N_c^{1/4} \Lambda_{QCD} \sim 1 - 2 \text{ GeV}$$

Cross section (black disk):

$$\sigma \sim \pi r_H^2$$



r_H : radius of horizon; at energy $E = \sqrt{s}$, $r_H \sim E^{1/(D-3)}$, so cross section

$$\sigma \sim s^{1/(D-3)} \sim s^{1/7} = s^{0.143}, \text{ for } D = 10$$

Increase energy $\Rightarrow r_H \nearrow \dots$ up to $r_H \sim R$ (saturation - Froissart behavior).

$D = 10$, so $R \sim E^{1/7}$ and for dimensional reasons,

$$E = M_P (M_P R)^7$$

Observed energy:

$$\hat{E} = \frac{R}{z_0} E = \Lambda_{QCD} (M_P R)^8 = N_c^2 \Lambda_{QCD} \sim 10 \text{ GeV}$$

For $\hat{E} \gtrsim N_c^2 \Lambda_{QCD}$, black hole “sees” IR brane.

- Use 10d shockwave model similar to Heisenberg's, based on...

[Kang and Nastase]

“The scattering process of two pointlike particles at CM energies in the order of Planck units or beyond, is very well calculable using known laws of physics, because graviton exchange dominates over all other interaction processes. At energies much higher than the Planck mass black hole production sets in, accompanied by coherent emission of real gravitons.”

- G. 't Hooft

⇒ For large r (4d distance), fixed z (interaction point), potential:

$$\Phi \sim \frac{1}{r^{11}}$$

cross section:

$$\sigma \sim s^\epsilon, \quad \epsilon = \frac{1}{11} = 0.0909 \dots$$

in agreement with experiment ($\epsilon_{\text{exp}} = 0.0933 \pm 0.0024$).

⇒ experimental evidence that $D = 10!$

As $\hat{E} \rightarrow \infty$, black hole is effectively on IR brane.

► to find σ , put point mass on IR brane with $m^2 = p^2 = s$.

Metric perturbation:

$$h_{00} \sim \frac{\sqrt{s}}{M_P^3 R} \frac{e^{-M_1 r}}{r}, \quad M_1 \sim \frac{1}{R} \quad (\text{mass of lightest KK mode})$$

observed mass: $RM/z_0 \sim \Lambda_{QCD} \sim \text{glueball mass.}$

black hole forms when $h_{00} \sim 1$, i.e.,

$$r_H \sim \frac{1}{M_1} \ln \frac{M_1 \sqrt{s}}{M_P^3 R}$$

confirming $r_H \sim \frac{1}{M_1} \sim R$. Cross section

$$\sigma \sim \pi r_H^2 \sim \frac{\pi}{M_1^2} \ln^2 \frac{M_1 \sqrt{s}}{M_P^3 R}$$

Froissart bound!

- ▶ Similar results for π scattering (bending of IR brane): $M_1 \rightarrow m_\pi$.
- ▶ Results can also be derived with shockwave model similar to Heisenberg's
[Kang and Nastase]

IMPORTANT CONCLUSION

In collision of two particles at energies

$$E \gg M_P$$

a black hole forms on IR brane with scattering cross section saturating the Froissart bound.

Dual statement (*observed*)

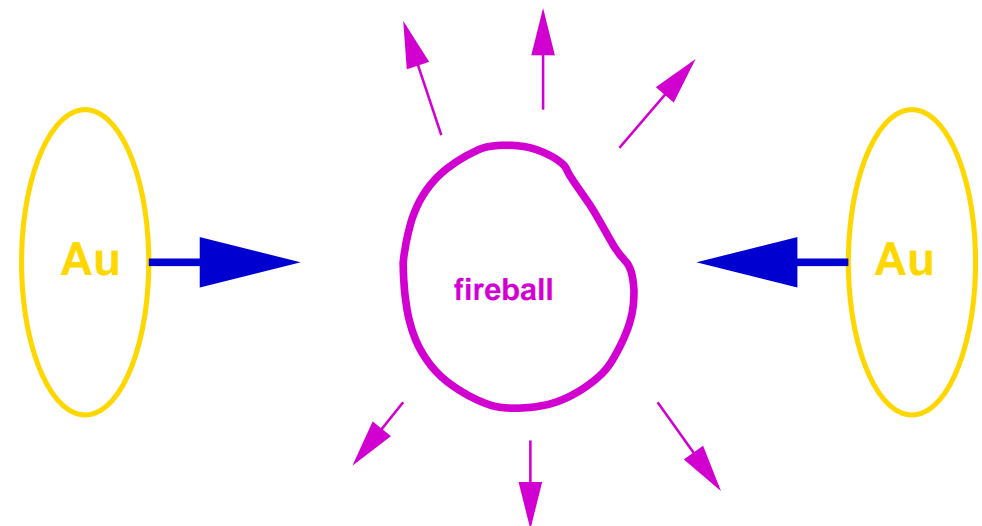
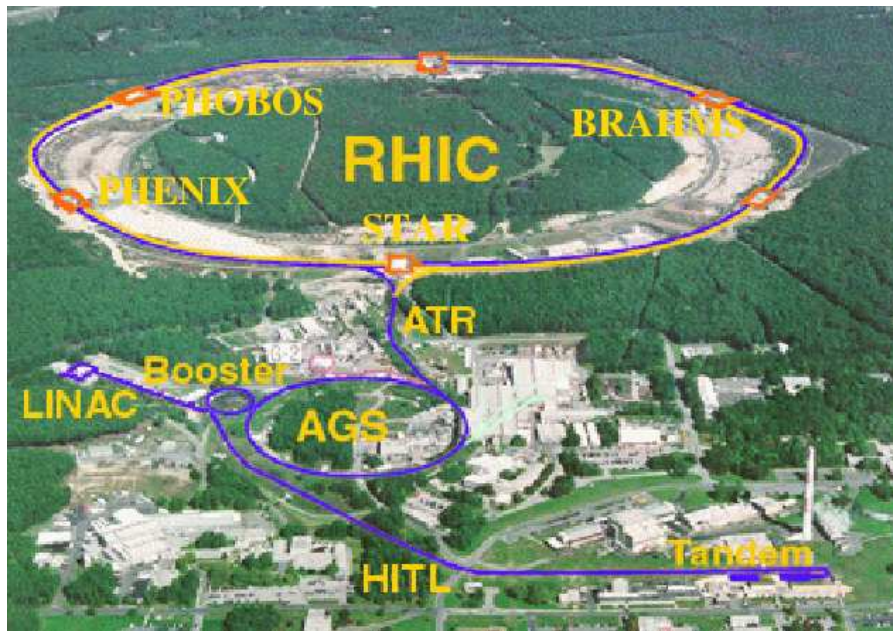
In collision of two *hadrons* at energies

$$\hat{E} \gg 10 \text{ GeV}$$

a *soliton* forms (pion field).

QUESTION: Has it been observed?

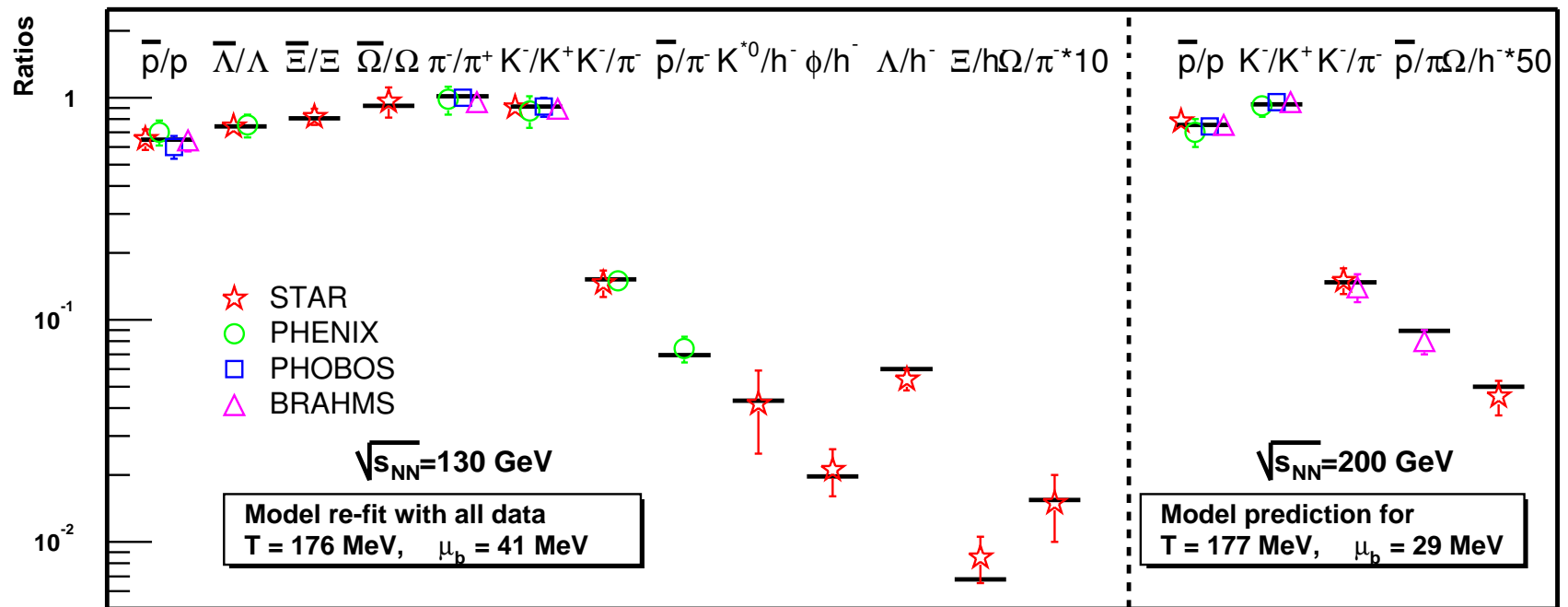
ANSWER: Yes, at RHIC!



- ▶ Two Au nuclei collide at 100 GeV per nucleon, so certainly $\hat{E} \gg 10 \text{ GeV}$.

- ▶ The soliton is the fireball observed
 - *color glass condensate (CGC)* instead of *quark gluon plasma (QGP)*; deconfined but strongly coupled phase
 - decays into thermally distributed particles at freeze-out temperature

$$T_{ch} \simeq 176 \text{ MeV}$$



[Braun-Munzinger, Magestro, Redlich and Stachel]

- hydronamic behavior with shear viscosity over entropy

$$\frac{\eta}{s} \sim 0.1 - 0.3$$

- *jet quenching* (absence of jets) observed - impossible in perturbative QCD.

TEMPERATURE

The CGC is the black hole interior.

- ▶ Black hole evaporates emitting Hawking radiation (gravitons) at temperature T_H .
- ▶ CGC decays into pions (dual gravitons).

It is hard to calculate the temperature of the black hole.

Hawking showed

$$T = \frac{\kappa}{2\pi}$$

where κ is surface gravity of horizon,

$$\kappa = \frac{1}{2} \left| \frac{\partial g_{00}}{\partial r} \right|_{r=r_H}$$

for 4d Kerr-Newman black holes.

Assume

$$T = a \frac{\kappa}{2\pi}$$

in our case (with a to be determined by an exact solution)

We have

$$g_{00} \simeq -1 + h_{00} = -1 + C \frac{e^{-M_1 r}}{r}$$

so

$$\kappa = \frac{1}{2} \left(M_1 + \frac{1}{r_H} \right) \simeq M_1$$

for $r_H \sim R \sim 1/M_1$. Temperature:

$$T = a \frac{M_1}{2\pi}$$

If this corresponds to the pion, then

$$M_1 = \langle m_\pi \rangle = \frac{1}{3} (m_{\pi^+} + m_{\pi^-} + m_{\pi^0})$$

If we set $a = 8$, then

$$T = \frac{4}{\pi} \langle m_\pi \rangle = 175.76 \text{ MeV}$$

► in agreement with experiment!

Viscosity

- ▶ In gravity,

$$\frac{\eta}{s} \geq \frac{1}{4\pi} \simeq 0.08$$

[Kovtun, Son and Starinets]

- ▶ bound saturated by black holes in type IIB supergravity
[Buchel and Liu]
- ▶ RHIC fireball is close to bound ($\eta/s \sim 0.1 - 0.3$).

Jet quenching

- ▶ *Information loss paradox*: only thermal radiation escapes from a black hole.
- ▶ No jets can be produced - they would carry information out of the dual black hole!

⇒ hard to probe interior of fireball.

Particle interactions in fireball are mostly pion-pion Coulomb interaction (Yukawa potential is almost Coulomb for $r \lesssim 1/m_\pi$).

- they correspond to Newtonian interactions in interior of black hole
- no ordinary black hole:
 - no singularity!
 - decays quickly

pion bound states \sim solar systems in black hole.

CONCLUSIONS

- We are beginning to understand non-perturbative QCD effects using string theory
- RHIC's fireball is a dual black hole predicted by string theory
[Nastase]
- RHIC and the LHC may probe black holes and provide information on string theory as well as non-perturbative QCD effects.
- Exciting future!